Section Day and Time:

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know or what you would do will get you partial credit!

1. (a) Calculate the following integral.

$$\int \frac{x^2 + x - 2}{3} \, dx$$

(b) Why does the same trick not work for

$$\int \frac{3}{x^2 + x - 2} \, dx?$$

(c) Calculate the integral

$$\int \frac{1}{x-1} - \frac{1}{x+2} \, dx.$$

(a) The best way to take this integral is to pull 1/3 out of the integral and be left with

$$\int \frac{x^2 + x - 2}{3} \, dx = \frac{1}{3} \int x^2 + x - 2 \, dx = \frac{1}{3} \left( \frac{1}{3} x^3 + \frac{1}{2} x^2 - 2x \right) + C.$$

The way I had intended for this to be done was to split the fraction to get

$$\int \frac{x^2 + x - 2}{3} \, dx = \int \frac{x^2}{3} + \frac{x}{3} - \frac{2}{3} \, dx = \frac{1}{9}x^3 + \frac{1}{6}x^2 - \frac{2}{3}x + C.$$

Of course, either way works.

- (b) If you did part (a) the second way, then what fails here is that we can't just split the denominator in the same way we split the numerator in part (a). If you did part (a) the first way, then the problem is that the variable is in the denominator and so the integral cannot be split across a sum like it was in part (a). These are the same problem, but we're thinking about them slightly differently.
- (c) Remembering that

$$\frac{d}{dx}\ln(u) = \frac{\frac{du}{dx}}{u},$$

we find

$$\int \frac{1}{x-1} - \frac{1}{x+2} \, dx = \int \frac{1}{x-1} \, dx - \int \frac{1}{x+2} \, dx = \ln|x-1| - \ln|x+2| + C.$$